Teaching Portfolio

Chris Dare

Contents

1 Teaching Statement

"'Obvious' is the most dangerous word in mathematics"

Eric Temple Bell

Despite its antiquity and wide prevalence across other fields, mathematics can be one of the most polarizing subjects for students. In the past, I always found it a bit odd that the most common response I got when telling someone I'm a mathematics Ph.D. student was an unabashed "I always hated math". The more I've engaged with my students in recent years, however, I've realized that this crowd is often trying to say "I always disliked my mathematics *courses*" and not "I dislike mathematics *as a subject*". The silver lining in the latter is that it is significantly easier address; though there are many factors that shape a students experience in a course, I believe that, as instructors, we play the most significant role in a student's success. As a consequence, it is my duty as an educator to constantly strive to improve aspects of my teaching so that students not only feel less deterred from mathematics, but find a sense of comfort in my classroom.

As a personal anecdote, pursuing an academic career in mathematics was never something in my sights until quite late. Like most people, mathematics was simply a check-mark to fulfill a general education requirement — at least until my first linear algebra course. The instructor, we were told, was fairly fresh out of graduate school and this would be his first full-time position. While that may give the impression he did not have the capacity for teaching a long-time professor would have, the opposite could not have been more true: our instructor, Mr. Margraff, had an excitement and enthusiasm for mathematics that became increasingly more contagious the closer he was in proximity to a chalkboard. What would normally be rudimentary exercises in most other linear algebra courses became colorful segues into the delicate nature of spaces in higher dimensions. Any misunderstanding in a student's question was met with an appreciative reply, which he saw as an opportunity to better explain a topic that may have been glossed over. It was shortly after this course that I went to the college of science at my university to declare a math major; to no mystery, this is due to an instructor who believed his students finding beauty in a subject was the best possible outcome of a course.

Unfortunately, one's journey through their math education rarely gives the same warm and inclusive feeling that Mr. Margraff gave. Even within the literature, a student will begin to notice that more and more details become omitted, often replaced with an unsatisfactory 'the proof is trivial' in its stead; sometimes authors will be more straightforward about their limitations, and give the infamous 'proof is left as an exercise to the reader'. While I do believe that practice is necessary to improve mathematical understanding, it becomes quickly apparent that math education can be pitted against those who do not see the forest through the trees on first glance. As a consequence, I have made it a primary objective in teaching to foster the idea that no detail is too trivial to withhold, no question is too rudimentary to carefully answer, and no misunderstanding should be assumed the fault of the student.

In effort to cultivate students curiosity, I have spent countless hours over the past several years meeting with students outside of my normal TA schedule to ensure that they have a safe space to ask questions they may not feel comfortable asking in front of peers. This has not only led to me meeting with current students quite regularly, but students enrolled in my previous classes as well — topics I have covered have ranged anywhere from high-school polynomial division to graduate-level Galois theory to programming-based problems. When not engaging directly with students, I still find myself spending the remaining hours of the day thinking of ways to better approach topics for students (examples of such are given in §4); though it can be a challenge exploring new teaching styles, I feel that it has both made me a better instructor and provided my students with new perspectives on the topics they learn.

Ultimately, teaching mathematics has been one of the most rewarding experiences in my educational career. Somehow finding myself come full circle, I now hope to give my students the same enthusiasm for mathematics that my linear algebra instructor, Mr. Margraff did, many years ago.

Though I cannot speak to whether every student has come out of my classes passionate about mathematics, at the very least I believe they walk away more confident in their mathematical abilities.

2 Diversity Statement

> "Mathematics knows no races or geographic boundaries; for mathematics, the cultural world is one country."
>
> *David Hilbert*

As an instructor, I believe that it is of the utmost importance to ensure that education is not just equally available to people of all races, genders, sexes, cultures, and beliefs, but *equitably* available. Specifically, it is vital to recognize that societal and socioeconomic disparities can, and have, lead to underrepresented students not getting same access to mathematics as their peers. Historically, mathematics has been dominated by institutions which disproportionately hire and cater to specific backgrounds, and the ripples of this imbalance are still felt today in mathematics departments across the world. Now more than ever, it is vital to foster a sense of belonging to underrepresented students to ensure that the knowledge and beauty of this subject does not become withheld from any group of people.

Among underrepresented groups, I believe that it is also absolutely necessary to ensure students with disabilities are given the means to participate and engage in mathematics at the same level as their peers. Just like any form of discrimination, ableism has no place in the academic setting — while this may seem like an obvious statement, there are still many indirect and passive forms of ableism which occur regularly in academia. This can include things like deflecting responsibility for accommodations to administrative departments, and refusing to incorporate accessible instructional resources into a curriculum. It becomes clear from both examples that, in order to ensure the success of these students, we as instructors must take a proactive role. To this end, I have spent a large portion of my collegiate education tutoring students with disabilities to not only help their academic careers, but inspire confidence in their strengths and abilities as well.

Lastly, I believe that it is important in today's political climate to address the importance of protecting the values and beliefs of all communities, and condemning any sort of censorship. The experiences of an individual or community should never be disregarded, and it is vital to ensure the voices of vulnerable communities are heard. Throughout my time as an instructor, I have been deeply committed to ensuring the opinions of every person in my classroom are valued, and no idea or trait is repressed.

3 Teaching History

Year	Quarter	Course	Instructor
2020	Winter		
	Spring		
	Summer		
	Fall	Math 34A — Calculus for Social Sciences	Daryl Cooper
2021	Winter	Math 6B — Vector Calculus II	Peter Garfield / Katy Craig
	Spring	Math 117 — Methods of Analysis	Katy Craig
	Summer	Math 4B — Differential Equations	Fabio Ricci
	Fall	Math 3B — Calculus with Applications II	Jea-Hyun Park
2022	Winter	Math 6B — Vector Calculus II	Zuhair Mullath
	Spring	Math 4B — Differential Equations	Gunhee Cho
	Summer		
	Fall	Math 4A — Linear Algebra	Peter Garfield
2023	Winter	Math 6A — Vector Calculus I	Marc Becker
	Spring	Math 3B — Calculus with Applications II	Peter Garfield
	Summer	Math 3B — Calculus with Applications II	Paige Hillen
	Fall	Math 6B — Vector Calculus II	Elie Abdo
2024	Winter	Math 8 — Transition to Higher Math	Wenchuan Tian
	Spring		
	Summer		
	Fall		

4 Sample Course Design

Though I have not yet had the opportunity to lead a course as instructor of record, I have been fortunate enough to exercise a good amount of freedom in several of my courses as a teaching assistant. This has ranged from designing my own quizzes to utilizing technology in unique ways in order to supplement students in their studies.

4.1 Sample Video Content / Visual Resources

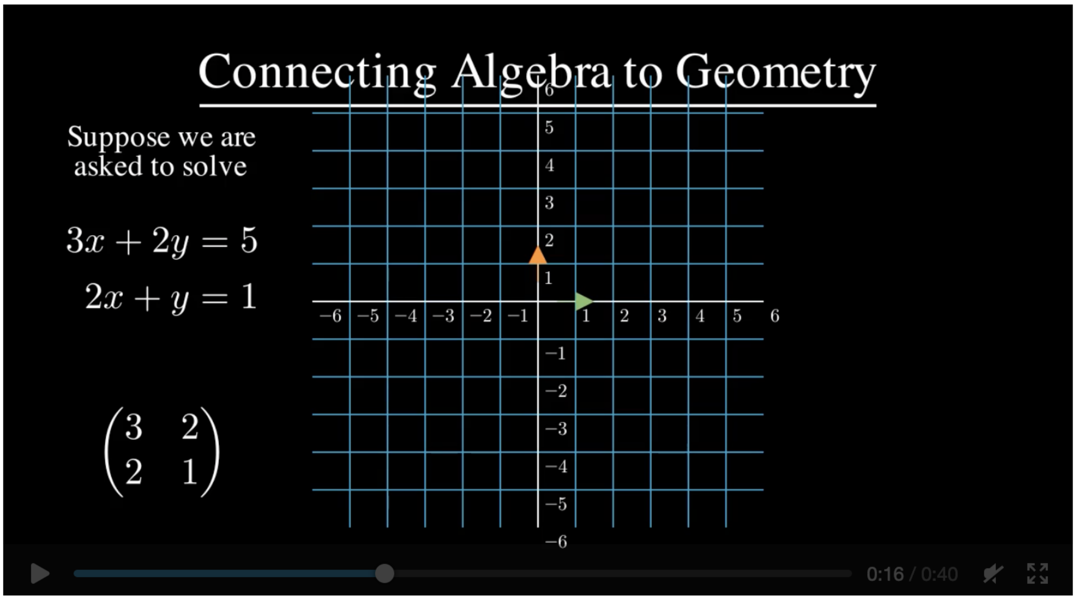
Growing up as a visual learner, I found it especially helpful to approach topics in mathematics based on what was happening geometrically. Unfortunately, the resources available become increasingly scarce as topics become more and more complex. For example, there is a wide variety of visual resources to help a student in a first year calculus course (e.g. Khan Academy, Brilliant, Professor Leonard to name a few); however, a student taking their first real analysis math course may quickly find that textbooks are essentially the only means of independent study. Though I do believe that parsing textbooks is an increasingly important skill the further one delves into mathematics, that does not mean it needs to be the *only* resource.

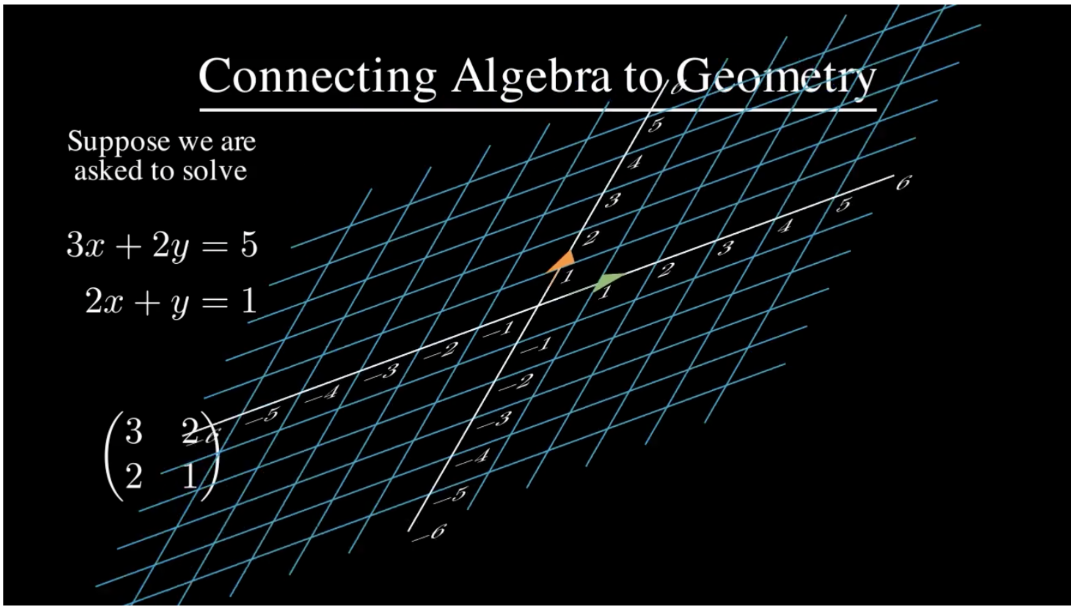
I am incredibly grateful to Grant Sanderson, creator of the popular mathematics YouTube Channel 3Blue1Brown, for making the Python library which he uses for animations (Manim) free and open source to the public in order to better provide educators with the tools to make engaging visual content. During the course of Summer 2022, I spent a large portion of my free time re-learning the basics of Python and becoming familiar with the Manim library in order to supplement my teaching during the 2022-2023 school year.

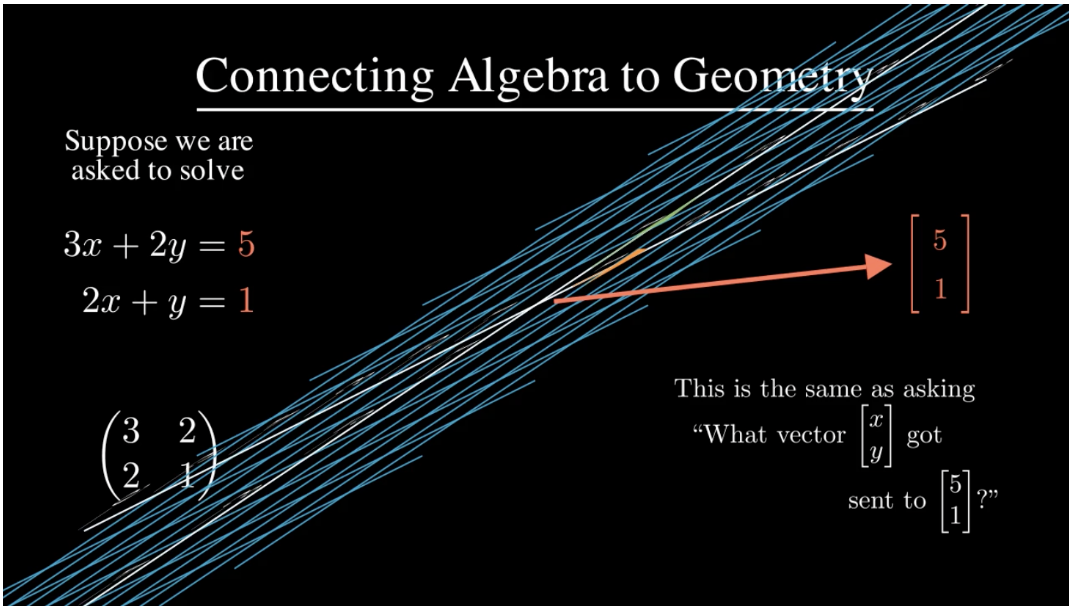
4.1.1 Math 4B: Linear Equations

In Fall 2022 I had the chance to TA for Math 4A (Linear Algebra) at UCSB under Professor Peter Garfield; as with most TA rôles, this came with the task of planning weekly 50-minute instruction sections. I believed it would be beneficial to provide students a short visual recap of the topics covered in lectures.

Below is an examples of one of the videos I would show at the beginning of every section:





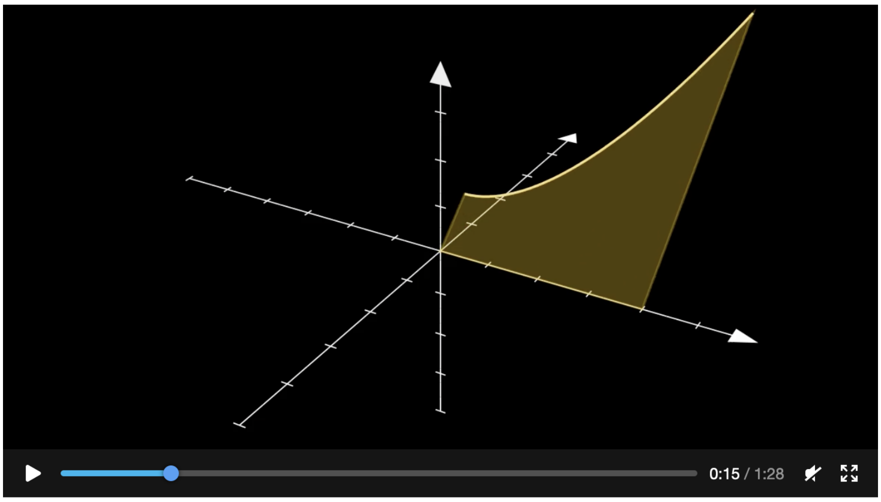


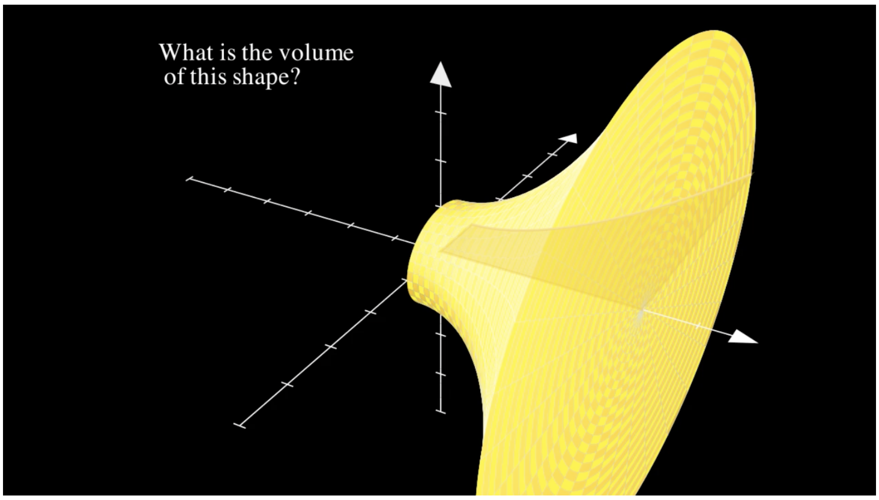
Since video (i.e. .MP4, .MOV) files are not able to be embedded in PDF files, I ask the reader to be generous in pretending the 3 frames provides an accurate approximation of the 40 second clip. However, anyone curious with how to render the scene in order to watch the full video is encouraged to use the code provided in Appendix A below.

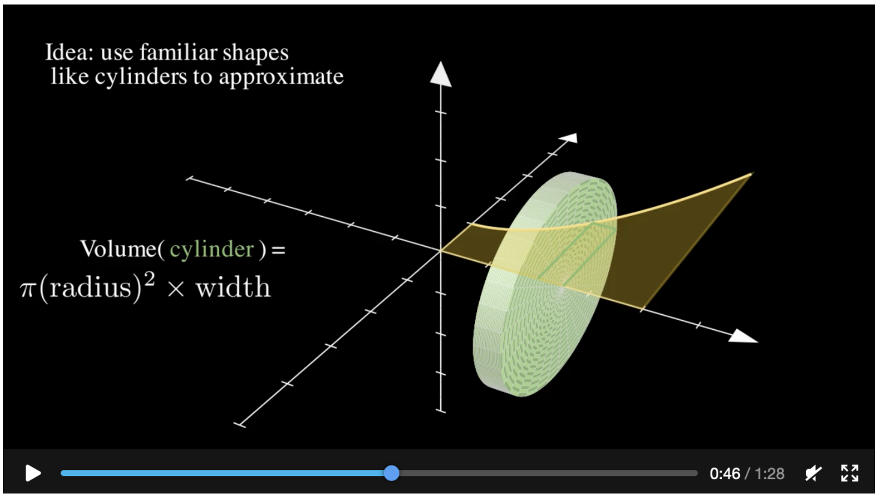
4.1.2 Math 3B: Volume / Surfaces of Revolution

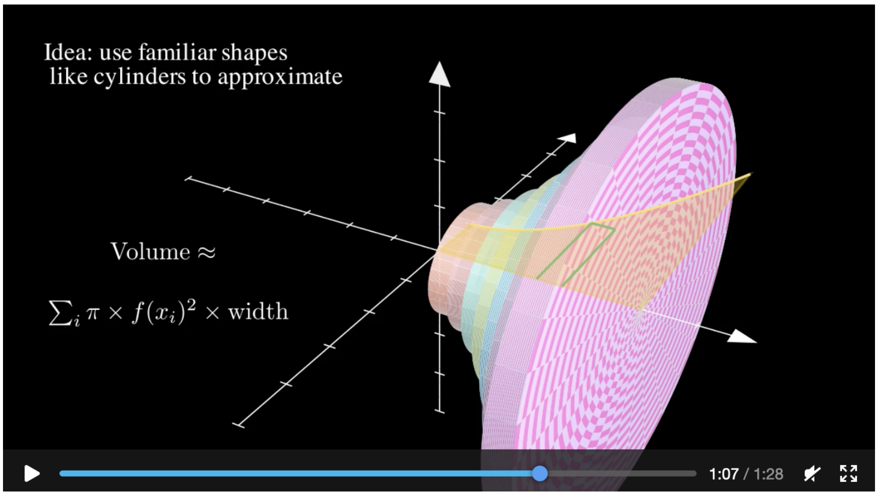
Fortunately, my efforts to provide a geometric intuition for topics at the beginning of each section found great success with my Math 4B students. Thus, it made sense to try to apply this strategy to future courses — for example, the following spring quarter I had the opportunity to once again TA for Professor Peter Garfield, now in the Math 3B (integral calculus). Though I mentioned visual resources for calculus courses are widely available online, I continued to spend several hours each week curating specifically tailored videos for the problems in the course.

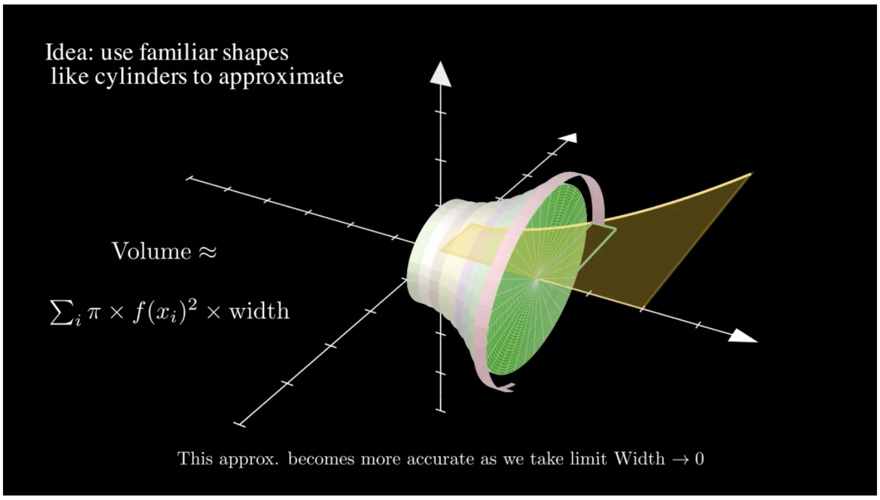
As an example, around the fourth week of instruction we began covering volume and surfaces of revolution — the latter subject is something I truly believe benefits from visual demonstration. Thus, I would begin certain problems during section by showing the students a geometric representation of what they were about to solve.











As before, the interested reader may consult Appendix B to reference the source code which was used to generate the 1:28 minute video.

4.2 Sample Interactive Content

In addition to providing videos to help student who benefit from visual learning techniques, it is important to address other learning styles as well. In my opinion, one of the most difficult learning styles to address in mathematics is kinaesthetic learning; since the vast majority of mathematics is conceptual, a good bit of creativity is required in order to keep these students engaged. While it may not be perfect, one solution I have found useful is giving students access to tools they can use to tinker with equations and variables (similar to Desmos for more advanced topics).

One of the primary ways I have done this is through the mathematical coding software Mathematica, which allows me to create interactive graphs that can then be uploaded to the WolframAlpha cloud servers via an API call:

```
    CloudDeploy[Manipulate[...]]
```

This was particularly useful in my Winter 2021 vector calculus course (taught by Professor Katy Craig), since it provided a means for students to dynamically interact with partial differential equations (PDEs) and observe the behavior of solutions as certain variables grow. For example, around week 9 of quarter we began discussing the wave equationm which is a tricky concept fundamental to the majority of quantum physics. In order to help with student's understanding, I gave a sample wave equation problem and uploaded the following interactive diagrams for students to utilize:

Wave equation coefficients:

In[124]:= $$\lambda[m\_, n\_] := c\sqrt{\left(\frac{m^2 \pi^2}{l^2} + \frac{n^2 \pi^2}{w^2}\right)};$$

$$\alpha[m\_, n\_] = \text{Simplify}\left[\frac{4}{l\,w} \int_0^l \int_0^w \text{v0}[x, y]\, \text{Sin}\left[\frac{m \pi x}{l}\right] \text{Sin}\left[\frac{n \pi y}{w}\right] dy\, dx, \{\text{Element}[n, \text{Integers}], \text{Element}[m, \text{Integers}]\}\right]$$

$$\beta[m\_, n\_] = \text{Simplify}\left[\frac{4}{l\,w\,\lambda[m, n]} \int_0^l \int_0^w \text{vt0}[x, y]\, \text{Sin}\left[\frac{m \pi x}{l}\right] \text{Sin}\left[\frac{n \pi y}{w}\right] dy\, dx, \{\text{Element}[n, \text{Integers}], \text{Element}[m, \text{Integers}]\}\right]$$

Out[125]= $$\frac{576\left(-1 + (-1)^m\right)\left(-1 + (-1)^n\right)}{m^3 n^3 \pi^6}$$

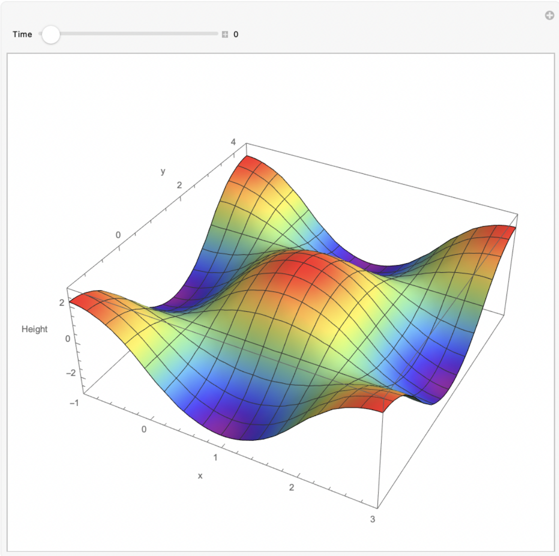
Out[126]= 0

Only sum first 5x5=25 terms to make computation quick(ish)

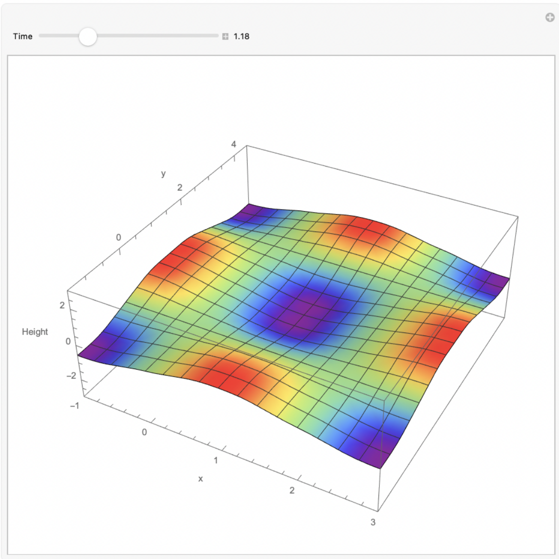
In[127]:= $$v[x\_, y\_, t\_] := \sum_{m=1}^{5}\sum_{n=1}^{5} (\alpha[m, n]\, \text{Cos}[\lambda[m, n]\, t] + \beta[m, n]\, \text{Sin}[\lambda[m, n]\, t])\, \text{Sin}\left[\frac{m \pi x}{l}\right] \text{Sin}\left[\frac{n \pi y}{w}\right];$$

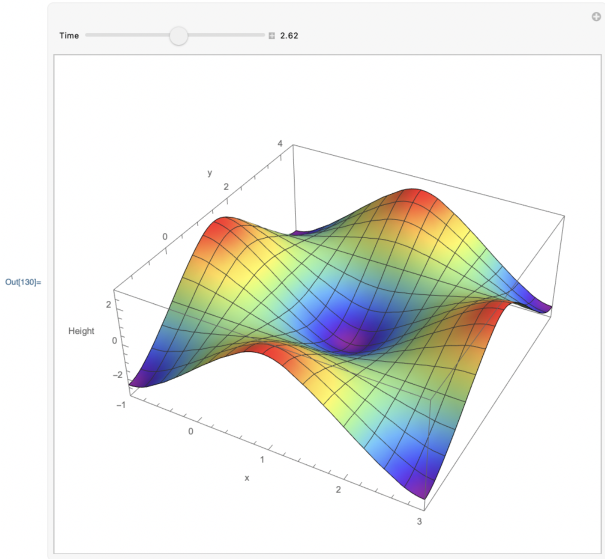
In[130]:=
```
Quiet@Manipulate[Plot3D[Release[v[x, y, t]], {x, -l / 2, l + l / 2}, {y, -w / 2, w + w / 2}, ImageSize → Large,
   PlotRange → {{-l / 2, l + l / 2}, {-w / 2, w + w / 2}, {-1 * Max[l, w], Max[l, w]}}, AxesLabel → {"x", "y", "Height"},
   PerformanceGoal → {"Quality", "Speed"}, ColorFunction → "Rainbow"], {{t, 0, "Time"}, 0, 5, 0.01, Appearance → "Labeled"}]
```

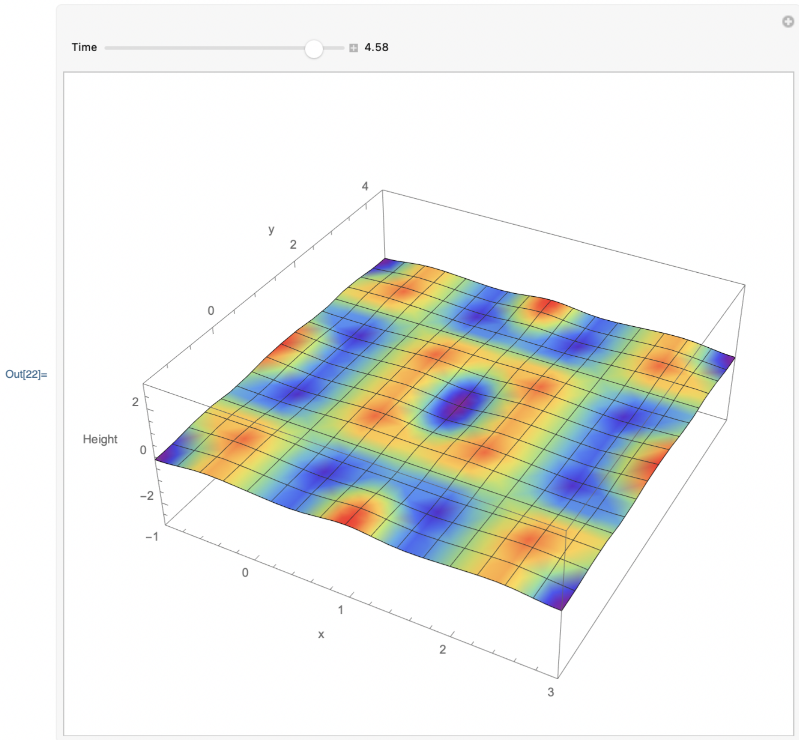
Out[130]=



Out[130]=







Unfortunately, it is significantly harder to provide the supplementary code used here in an appendix since Mathematica heavily utilizes the markdown language (which requires a handful of libraries to convert into LaTeX).

4.3 Sample Quiz

Name: ______________________________

Student Number: ______________________________

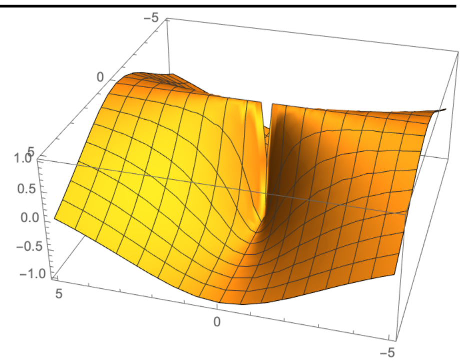
Write all steps clearly in order to get any partial credit. No calculators, outside notes, or collaboration are allowed. By signing your name, you agree to adhere to and uphold the UCSB Academic Integrity statement.

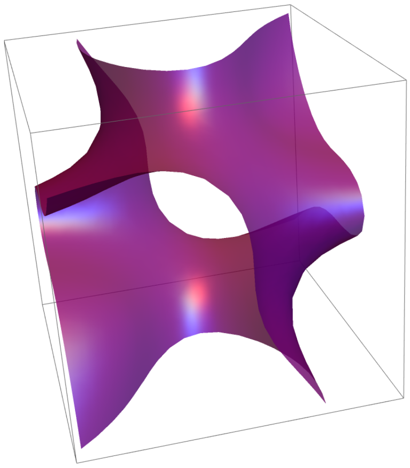
Distribution of Marks

Problem	Points	Score
1	5	
2	5	
3	5	
4	5	
Total	20	

---

(1) [5 Points] Consider the "pinched plane" given by the equation $f(x, y) = \frac{x^2-y^2}{x^2+y^2}$. Using your geometric intuition based off the following picture, justify whether the limit $\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2}$ exists.





(2) [5 Points] One of the most significant kinds of shapes to theoretical physicists is something known as 'Calabi-Yau Manifolds' — a slice of one can be given by the equation $xz^3 + 2y^2z^2 - yx^3 = 2$. Find what the tangent plane is at the point $(x, y, z) = (0, 1, 1)$

(3) [5 Points] Given $f(x, y) = e^{5-2x+3y}$, use the point $(x_0, y_0) = (4, 1)$ to (linearly) approximate the value of $f(4.1, 0.9)$.

(4) [5 Points] Consider the function $g(v, w) = \langle ve^w, e^v - w, we^2 \rangle$. What is the Jacobian matrix for g?

5 Student Feedback

5.1 Course Evaluations

5.1.1 Evaluation 1

ESCI ONLINE SURVEY STATISTICS 3/30/23

Note that the Campus and Departmental Norms for this Survey are based ONLY on other ESCI Online Courses.
Due to the different method of data collection, these Norms do not include ESCI Surveys collected by the paper response forms.

End of Winter Quarter 2023 -- ESCI Online

Department and Campus Norms taken over time span: Spring Quarter 2018 - Winter Quarter 2023

Abbrv: **MATH** Instructor: **DARE C E** Rank: **Teaching Assistant** Course: **MATH 6A 0105** Type: **Discussion**
Department: **MATHEMATICS** Course Enrollment: **26**

Guidelines for "Interpreting ESCI Data" and a description of the "Report Output" can be found at http://oic.id.ucsb.edu/esci.
NOTICE: Please examine these evaluations upon receipt and immediately report any suspected errors to: ESCI Office, Instructional Development, 1130 Kerr Hall (x4278) or (id-esci@ucsb.edu)

(3260) **1.** How enthusiastic was the TA about teaching?

(a) Very enthusiastic
(b) Quite enthusiastic
(c) Somewhat enthusiastic
(d) Slightly enthusiastic
(e) Not enthusiastic

Response weighting:	1	2	3	4	5	Blank	Total	Total		
*NOTE: Each Student Response=8%	(a)	(b)	(c)	(d)	(e)	Response	Students	Courses	Mean	Median
***--> This COURSE current quarter**	**58%**	**33%**	**8%**			**0**	**12**	**1**	**1.5**	**1.0**
Student-weighted Norms (UG students)										
Dept MATH TAs current qtr	51%	29%	13%	4%	4%	3	1671	196	1.8	1.0
Dept MATH TAs over time	49%	29%	15%	4%	3%	95	29721	2490	1.8	2.0
Campus TAs over time	49%	29%	15%	4%	3%	95	29721	2490	1.8	2.0

(3261) **2.** Rate the clarity of the TA's explanations.

(a) Excellent (b) Good (c) Fair (d) Poor (e) Very poor

Response weighting:	1	2	3	4	5	Blank	Total	Total		
*NOTE: Each Student Response=8%	(a)	(b)	(c)	(d)	(e)	Response	Students	Courses	Mean	Median
***--> This COURSE current quarter**	**92%**	**8%**				**0**	**12**	**1**	**1.1**	**1.0**
Student-weighted Norms (UG students)										
Dept MATH TAs current qtr	57%	26%	12%	3%	2%	3	1671	196	1.7	1.0
Dept MATH TAs over time	52%	30%	13%	4%	2%	116	29721	2490	1.7	1.0
Campus TAs over time	52%	30%	13%	4%	2%	116	29721	2490	1.7	1.0

DARE C E
MATH 6A 0105 (T 800 -850 HSSB 1206) Survey Number: **411722** Page 1

ESCI ONLINE SURVEY STATISTICS 3/30/23

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End of Winter Quarter 2023 -- ESCI Online

Department and Campus Norms taken over time span: Spring Quarter 2018 - Winter Quarter 2023

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Department: **MATHEMATICS** Course Enrollment: **26**

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(3262) **3.** How available was the TA to help students?

(a) Very available
(b) Reasonably available
(c) Sometimes available
(d) Hardly available
(e) Not available

(f) Not applicable

Response weighting:	1	2	3	4	5	0	Blank	Total	Total		
*NOTE: Each Student Response=8%	(a)	(b)	(c)	(d)	(e)	(f)	Response	Students	Courses	Mean	Median
***--> This COURSE current quarter**	**75%**	**17%**				**8%**	**0**	**12**	**1**	**1.2**	**1.0**
Student-weighted Norms (UG students)											
Dept MATH TAs current qtr	58%	29%	7%	2%		4%	2	1671	196	1.5	1.0
Dept MATH TAs over time	54%	30%	7%	2%	1%	6%	116	29721	2490	1.6	1.0
Campus TAs over time	54%	30%	7%	2%	1%	6%	116	29721	2490	1.6	1.0

(3263) **4.** Rate the TA's preparation for conducting section.

(a) Very prepared
(b) Quite prepared
(c) Somewhat prepared
(d) Slightly prepared
(e) Not prepared

Response weighting:	1	2	3	4	5	Blank	Total	Total		
*NOTE: Each Student Response=8%	(a)	(b)	(c)	(d)	(e)	Response	Students	Courses	Mean	Median
***--> This COURSE current quarter**	**92%**	**8%**				**0**	**12**	**1**	**1.1**	**1.0**
Student-weighted Norms (UG students)										
Dept MATH TAs current qtr	60%	27%	9%	2%	2%	14	1671	196	1.6	1.0
Dept MATH TAs over time	57%	29%	10%	2%	2%	277	29721	2490	1.6	1.0
Campus TAs over time	57%	29%	10%	2%	2%	277	29721	2490	1.6	1.0

DARE C E
MATH 6A 0105 (T 800 -850 HSSB 1206) Survey Number: **411722** Page 2

ESCI ONLINE SURVEY STATISTICS 3/30/23

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End of Winter Quarter 2023 -- ESCI Online

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Department: **MATHEMATICS** Course Enrollment: **26**

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(3264) **5.** Rate your TA's overall performance in helping you master the material.

(a) Excellent (b) Good (c) Fair (d) Poor (e) Very poor

Response weighting:	1	2	3	4	5					
*NOTE: Each Student Response=8%	(a)	(b)	(c)	(d)	(e)	Blank Response	Total Students	Total Courses	Mean	Median
***--> This COURSE current quarter**	**92%**	**8%**				**0**	**12**	**1**	**1.1**	**1.0**
Student-weighted Norms (UG students)										
Dept MATH TAs current qtr	56%	28%	10%	4%	2%	9	1671	196	1.7	1.0
Dept MATH TAs over time	50%	30%	13%	4%	2%	188	29721	2490	1.8	1.0
Campus TAs over time	50%	30%	13%	4%	2%	188	29721	2490	1.8	1.0

(3265) **6.** The TA created a classroom environment in which students felt ________ asking questions.

(a) Very comfortable
(b) Quite comfortable
(c) Somewhat comfortable
(d) Slightly comfortable
(e) Not comfortable

Response weighting:	1	2	3	4	5					
*NOTE: Each Student Response=8%	(a)	(b)	(c)	(d)	(e)	Blank Response	Total Students	Total Courses	Mean	Median
***--> This COURSE current quarter**	**75%**	**17%**	**8%**			**0**	**12**	**1**	**1.3**	**1.0**
Student-weighted Norms (UG students)										
Dept MATH TAs current qtr	60%	26%	9%	3%	2%	8	1671	196	1.6	1.0
Dept MATH TAs over time	57%	27%	11%	3%	2%	241	29721	2490	1.7	1.0
Campus TAs over time	57%	27%	11%	3%	2%	241	29721	2490	1.7	1.0

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Department: **MATHEMATICS** Course Enrollment: **26**

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NOTICE: Please examine these evaluations upon receipt and immediately report any suspected errors to: ESCI Office, Instructional Development, 1130 Kerr Hall (x4278) or (id-esci@ucsb.edu)

(3266) **7. Please write any comments you have regarding this TA's instruction.**

Chris was always very open to questions during section and answered them excitedly and thoroughly. He was also very approachable during office hours and math lab. All his explanations really helped, and I enjoyed going to section because each time, it would really solidify the information we had been learning the prior week.

He usually explained material better than the professor and I felt that section was very beneficial to this class because of the TA I had. He clearly explained concepts and the reasoning behind them, which was very effective, and seemed very knowledgeable on whatever we asked him in section, whether is was on the material or the course in general.

Extremely helpful sections. Mainly helped me understand things conceptually which was wonderful.

Really good at clarifying concepts explained in lecture, working through examples with him in section was incredibly helpful.

I was in his morning section and I know nobody ever responded since we were all tired at 8am but Chris always welcomed us with a Good morning and was ready to teach the lecture material. He did a great job considering he was the only TA and quickly graded the section's weekly quizzes. Chris was a very good TA and should keep up what he's doing if he continues to be a TA.

Chris was a great instructor, with very aware and informative sections

---

You save this class thank you so much

5.1.2 Evaluation 2

ESCI ONLINE SURVEY STATISTICS

3/30/23

Note that the Campus and Departmental Norms for this Survey are based ONLY on other ESCI Online Courses.
Due to the different method of data collection, these Norms do not include ESCI Surveys collected by the paper response forms.

End of Winter Quarter 2023 -- ESCI Online

Department and Campus Norms taken over time span: Spring Quarter 2018 - Winter Quarter 2023

Abbrv: **MATH** Instructor: **DARE C E** Rank: **Teaching Assistant** Course: **MATH 6A 0103** Type: **Discussion**
Department: **MATHEMATICS** Course Enrollment: **25**

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(3260) **1.** How enthusiastic was the TA about teaching?

(a) Very enthusiastic
(b) Quite enthusiastic
(c) Somewhat enthusiastic
(d) Slightly enthusiastic
(e) Not enthusiastic

Response weighting: *NOTE: Each Student Response=25%	1 (a)	2 (b)	3 (c)	4 (d)	5 (e)	Blank Response	Total Students	Total Courses	Mean	Median
***--> This COURSE current quarter**	**100%**					**0**	**4**	**1**	**1.0**	**1.0**
Student-weighted Norms (UG students)										
Dept MATH TAs current qtr	51%	29%	13%	4%	4%	3	1671	196	1.8	1.0
Dept MATH TAs over time	49%	29%	15%	4%	3%	95	29721	2490	1.8	2.0
Campus TAs over time	49%	29%	15%	4%	3%	95	29721	2490	1.8	2.0

(3261) **2.** Rate the clarity of the TA's explanations.

(a) Excellent (b) Good (c) Fair (d) Poor (e) Very poor

Response weighting: *NOTE: Each Student Response=25%	1 (a)	2 (b)	3 (c)	4 (d)	5 (e)	Blank Response	Total Students	Total Courses	Mean	Median
***--> This COURSE current quarter**	**100%**					**0**	**4**	**1**	**1.0**	**1.0**
Student-weighted Norms (UG students)										
Dept MATH TAs current qtr	57%	26%	12%	3%	2%	3	1671	196	1.7	1.0
Dept MATH TAs over time	52%	30%	13%	4%	2%	116	29721	2490	1.7	1.0
Campus TAs over time	52%	30%	13%	4%	2%	116	29721	2490	1.7	1.0

ESCI ONLINE SURVEY STATISTICS

3/30/23

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(3262) **3.** How available was the TA to help students?

(a) Very available
(b) Reasonably available
(c) Sometimes available
(d) Hardly available
(e) Not available

(f) Not applicable

Response weighting: *NOTE: Each Student Response=25%	1 (a)	2 (b)	3 (c)	4 (d)	5 (e)	0 (f)	Blank Response	Total Students	Total Courses	Mean	Median
***--> This COURSE current quarter**	**100%**						**0**	**4**	**1**	**1.0**	**1.0**
Student-weighted Norms (UG students)											
Dept MATH TAs current qtr	58%	29%	7%	2%		4%	2	1671	196	1.5	1.0
Dept MATH TAs over time	54%	30%	7%	2%	1%	6%	116	29721	2490	1.6	1.0
Campus TAs over time	54%	30%	7%	2%	1%	6%	116	29721	2490	1.6	1.0

(3263) **4.** Rate the TA's preparation for conducting section.

(a) Very prepared
(b) Quite prepared
(c) Somewhat prepared
(d) Slightly prepared
(e) Not prepared

Response weighting: *NOTE: Each Student Response=25%	1 (a)	2 (b)	3 (c)	4 (d)	5 (e)	Blank Response	Total Students	Total Courses	Mean	Median
***--> This COURSE current quarter**	**100%**					**0**	**4**	**1**	**1.0**	**1.0**
Student-weighted Norms (UG students)										
Dept MATH TAs current qtr	60%	27%	9%	2%	2%	14	1671	196	1.6	1.0
Dept MATH TAs over time	57%	29%	10%	2%	2%	277	29721	2490	1.6	1.0
Campus TAs over time	57%	29%	10%	2%	2%	277	29721	2490	1.6	1.0

ESCI ONLINE SURVEY STATISTICS 3/30/23

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End of Winter Quarter 2023 -- ESCI Online

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(3264) **5.** Rate your TA's overall performance in helping you master the material.

(a) Excellent (b) Good (c) Fair (d) Poor (e) Very poor

Response weighting:	1	2	3	4	5	Blank	Total	Total		
*NOTE: Each Student Response=25%	(a)	(b)	(c)	(d)	(e)	Response	Students	Courses	Mean	Median
***--> This COURSE current quarter**	**100%**					**0**	**4**	**1**	**1.0**	**1.0**
Student-weighted Norms (UG students)										
Dept MATH TAs current qtr	56%	28%	10%	4%	2%	9	1671	196	1.7	1.0
Dept MATH TAs over time	50%	30%	13%	4%	2%	188	29721	2490	1.8	1.0
Campus TAs over time	50%	30%	13%	4%	2%	188	29721	2490	1.8	1.0

(3265) **6.** The TA created a classroom environment in which students felt _________ asking questions.

(a) Very comfortable
(b) Quite comfortable
(c) Somewhat comfortable
(d) Slightly comfortable
(e) Not comfortable

Response weighting:	1	2	3	4	5	Blank	Total	Total		
*NOTE: Each Student Response=25%	(a)	(b)	(c)	(d)	(e)	Response	Students	Courses	Mean	Median
***--> This COURSE current quarter**	**100%**					**0**	**4**	**1**	**1.0**	**1.0**
Student-weighted Norms (UG students)										
Dept MATH TAs current qtr	60%	26%	9%	3%	2%	8	1671	196	1.6	1.0
Dept MATH TAs over time	57%	27%	11%	3%	2%	241	29721	2490	1.7	1.0
Campus TAs over time	57%	27%	11%	3%	2%	241	29721	2490	1.7	1.0

ESCI ONLINE SURVEY STATISTICS 3/30/23

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(3266) **7. Please write any comments you have regarding this TA's instruction.**

Very very good at teaching/summarizing lecture content, quizzes were always helpful, practice midterms were always very beneficial. Very understanding, approachable, respectful.

Chris, you're one of the best math instructors I've ever had in your patience and approach. Please don't stop teaching, please survive academia and make it out the same amazing teacher you already are :)

Chris did an excellent job explaining previous weeks' concepts in a more digestible and easier-to-understand format.

Could not have wished for a more approachable, helpful, understanding, and cooperative TA. Listened to problems whether they were administrative or content related. 10/10

5.2 Student Emails / Additional Correspondence

Thank you! Inbox ×

[redacted]@umail.ucsb.edu
to Chris
Thu, Dec 14, 2023, 10:14 PM

Dear Chris,

Thank you for a great quarter of Math 6B! Sections were great and I appreciated the effort you put into our class.

Best of luck with your geometry research and have a great winter break!

[redacted]

Thank You Letter Inbox ×

[redacted]@umail.ucsb.edu
to me
Wed, Dec 13, 2023, 3:51 PM

Hello Chris,

Just want to say a big thank you for being such an excellent TA this quarter. It would be impossible to get an A+ without your help and guidance. I appreciate all the reviews of important topics and problem-solving skills. Wish you have a great winter break and hope to see you in future courses.

Best,
[redacted]

[redacted] <[redacted]@ucsb.edu>
to Chris
Thu, Mar 30, 2023, 11:48 AM

Hi Chris,

I hope you are doing well!

I just wanted to thank you for all you have done this quarter. You were my TA these past winter and fall quarters in 4A and 6A and the way you ran your section in both always seemed to go above and beyond what was required of you. I have had some trouble seeing the point in some of my other sections, so thank you for running yours in a way that enhanced the lecture, instead of just repeating it. I found you explained the material often better than the professor, and the models you made really aided in my understanding. So, I just wanted to say thank you for the time and effort you put into your job and that I really appreciate it.

Have a great rest of your break!

Best,
[redacted]

[redacted]@umail.ucsb.edu
to cdare
Thu, Mar 10, 2022, 8:44 PM

Thank you so much for your very detailed response, Chris! I really really appreciate it and all your efforts this quarter! You're the most responsible and considerate TA I've ever met!!

<cdare@umail.ucsb.edu>于2022年3月10日 周四下午6:59写道：

6 Mentoring of Undergraduate Research

It was a pleasure to be a part of the UCSB Directed Reading Program (DRP) as a mentor to undergraduate mathematical research during the 2022-2023 school year. With a focus towards algebraic and complex geometry, I had the chance to work with a student on Hodge theory in the hope of covering the basics of Hodge theory and why it is important to geometers. We spent 14 weeks covering the following topics:

(1) Smooth manifolds:
   - Topological spaces
   - Homeomorphisms and open charts
   - Differential forms and the (co)tangent bundle
   - Complex manifolds and complex structures

(2) Cohomology on manifolds:
   - De Rham cohomology
   - Dolbeault cohomology

(3) Basic Hodge Theory:
   - The Hodge diamond
   - Hodge structures
   - Correspondence between Hodge structures of weight 1 and tori

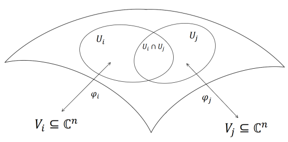
Following the 14 week instruction period, my student had the opportunity to create a poster based on the culmination of their research and present it to the UCSB mathematics faculty.

MANIFOLDS, COHOMOLOGIES, AND HODGE STRUCTURES

Hespos Goodman
University of California, Santa Barbara

COMPLEX MANIFOLDS

An n-dimensional **complex manifold** is a topological space that is locally isomorphic to $\mathbb{C}^n$. This means manifolds can take arbitrary, and often extremely complicated, forms on a global scale, but "zooming in" allows us to study their local properties with relative ease. This construction is defined by an **atlas** of open sets $(U_i)_{i\in M}$ that cover our manifold, each with a **chart** (φ) that that links it to $\mathbb{C}$



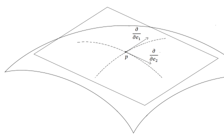
In order for this construction to be useful. It must guarantee continuity of functions on the surface of our manifold. This is achieved through requiring that our charts (φ_i) be *holomorphic (analytic) diffeomorphisms*, and requiring that, on the intersection, the composition $\varphi_i^{-1} \circ \varphi_j$ is a smooth map.

TANGENT BUNDLES

Now that we know what the surface of a manifold looks like, we can begin talking about what happens along that surface. At any particular point p we define T_pM, the **tangent space** at that point. This space in generated by the tangent vectors (at p) of every curve on our manifold that passes through p. As usual, this is equivalent to using the partial derivatives of our chart with respect to the basis vectors in V_i at the preimage of our point,

$$T_pM = \left\langle \frac{\partial}{\partial e_1}(\varphi_i)\bigg|_{\varphi_i^{-1}(p)}, \frac{\partial}{\partial e_2}(\varphi_i)\bigg|_{\varphi_i^{-1}(p)}, \cdots, \frac{\partial}{\partial e_{2n}}(\varphi_i)\bigg|_{\varphi_i^{-1}(p)} \right\rangle = \left\langle \frac{\partial}{\partial e_1}, \frac{\partial}{\partial e_2}, \cdots, \frac{\partial}{\partial e_{2n}} \right\rangle$$

The notation on the right hand side is less formal, but is permissible in the local (U_i) frame. Note that this basis is isomorphic to $\mathbb{R}^{2n}$ under $\frac{\partial}{\partial e_1} \leftrightarrow e_1$. Which yields the usual understanding of a tangent space, depicted for a 2-(real)-dimensional manifold on the right.



The tangent space is specific to each individual point, because it relies on evaluating the partial derivative at the unique (restricted to U_i) preimage of p. In order to address the manifold at large, we can define a **tangent bundle** (TM) which is the set of all pairs of points (p), and vectors in that point's tangent space.

$$TM = \{(p, \vec{v}) \mid p \in M, \vec{v} \in T_pM\}$$

Naturally, there are a LOT of vectors in the tangent space of any particular point. The **(tangent) vector field** (ξ) provides us a method for selecting one of these vectors, given a particular point.

$$\begin{aligned} \xi := M &\mapsto TM \\ p &\mapsto (p, \vec{v}) \end{aligned}$$

For the purposes of integration, we want to remember which point each of these vectors comes from. This is why its essential for the vector field to map to the tangent bundle rather than a particular tangent space.

COTANGENT BUNDLES

Using our definitions of tangent spaces, bundles, and fields, we will define cotangent spaces, bundles, and fields. A covector (ω)(also called a 0-form, or a linear functional) is a function that takes in a vector and outputs a scalar.

$$\omega := \vec{v} \mapsto z$$

Naturally a **cotangent vector** is a covector who's domain is the tangent space (at a point), so we can be sure that it intakes tangent vectors. Applying what we know about tangent spaces, we can see that the **cotangent space** should be the space of all cotangent vectors.

$$T_p^*M = \{\omega | \omega : T_pM \mapsto \mathbb{C}\}$$

COTANGENT BUNDLES (CONT.)

Here we use the notation for the dual of the tangent space since thats exactly what the cotangent space is! It is the set of all maps(covectors) from the tangent space to the underlying field($\mathbb{C}$ in our case). In light of this, we can define a basis for the cotangent space, with the conventional linear functional basis of a dual space

$$T_p^*M = \langle de_1^p, de_2^p, \cdots, de_{2n}^p \rangle, \quad de_i^p\left(\frac{\partial}{\partial e_j}\bigg|_p\right) = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$$

Similarly, the **cotangent bundle** is the set of all point-cotangent vector pairs

$$T^*M = \{(p, \omega) | p \in M, \omega \in T_p^*M\}$$

Again, this is the dual of the tangent bundle
Finally, a **covector field** is analogous to a vector field. It is a map that, given a point, provides a covector in the cotangent space of that point

$$\begin{aligned} \alpha := M &\mapsto T^*M \\ p &\mapsto (p, \omega) \end{aligned}$$

When we require this map to be smooth, we realize this "covector field" as a section of the cotangent bundle, or a **differential one form**

DIFFERENTIAL 1-FORMS AND EXTERIOR DERIVATIVES

Differential 1-forms are functions that are nearly equivalent to covector fields, the main difference is that we allow them to intake a point AND a vector (i.e. a vector field), so their output becomes a point-scalar pair . Differential forms are written

$$\alpha(p, \vec{v}) = \left(p, \sum_{i=1}^{2n} f_i(p) de_i^p(\vec{v})\right) = \left(p, f_1(p)de_1^p(\vec{v}) + f_2(p)de_2^p(\vec{v}) + \cdots + f_{2n}(p)de_{2n}^p(\vec{v})\right)$$

So, in the particular case where $\vec{v} = \frac{\partial}{\partial e_i}$ that we achieve

$$\alpha\left(p, \frac{\partial}{\partial e_i}\bigg|_p\right) = 0 + \cdots + f_i(p)de_i^p\left(\frac{\partial}{\partial e_i}\bigg|_p\right) + \cdots + 0 = (p, f_i(p))$$

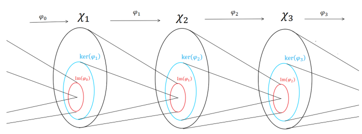
Inspecting the second term, evaluation of α at a point allows us to "measure" the value of α in the $\frac{\partial}{\partial e_i}\big|_p$ direction. So, summing α along a curve is equivalent to integrating f_i with respect to e_i. More generally, evaluating along some vector field, ξ, allows us to integrate along our entire manifold (with respect to ξ).
Thus, 1-forms are the tools we use in every one dimensional integral. We can use the **exterior derivative** to achieve 2-forms, which allow us to integrate area, and eventually m-forms, which measure m-dimensional oriented density.
The exterior derivative, d, asks us to differentiate each of our f_is with respect to each e_j^p and to note that differentiation in the result

$$d(\alpha) = \sum_{j=1}^{2n}\sum_{i=1}^{2n} \frac{\partial f_i}{\partial e_j} de_j^p \wedge de_i^p$$

The wedge product ($\wedge$) here is a complicated algebraic structure that explicitly outlines how to evaluate the vector part of our input.

COHOMOLOGIES

In order to better understand the properties of a certian manifold, it can be helpful to understand how differential forms of change as we differentiate them. A sequence of groups (and maps from one group to the next) is called *exact* if $\ker \phi_i = \text{im}\phi_{i-1}$



Inspecting the quotient group $ker(\varphi_i) / Im(\varphi_{i-1})$ allows us to measure how far a sequence is from being exact. This is the premise behind cohomology.

THE DOLBEAULT COHOMOLOGY

Since we are working with a complex manifold. We can chose a convenient basis to address our tangent and cotangent spaces

$$\langle z_1, \overline{z_1}, \cdots, z_n, \overline{z_n}, \rangle$$

This choice of basis leads to a method for splitting the exterior derivative

$$d = \partial + \overline{\partial}$$

Where ∂ takes the partial derivatives with respect to the complex basis $\langle z_1, z_2, \cdots, z_n \rangle$, and $\overline{\partial}$ takes the partial derivatives with respect to the complex conjugate basis $\langle \overline{z_1}, \overline{z_2}, \cdots, \overline{z_n} \rangle$. Using these operators, we can construct a cohomology in two directions. Begining with $\Omega^{0,0}$ the space of 0-forms (covectors) we construct,

$$\begin{array}{ccccccc}
\Omega^{0,0} & \xrightarrow{\partial_{0,0}} & \Omega^{1,0} & \xrightarrow{\partial_{1,0}} & \Omega^{2,0} & \xrightarrow{\partial_{2,0}} & \\
\downarrow \bar{\partial}_{0,0} & & \downarrow \bar{\partial}_{1,0} & & \downarrow \bar{\partial}_{2,0} & & \\
\Omega^{0,1} & \xrightarrow{\partial_{0,1}} & \Omega^{1,1} & \xrightarrow{\partial_{1,1}} & \Omega^{2,1} & \xrightarrow{\partial_{2,1}} & \\
\downarrow \bar{\partial}_{0,1} & & \downarrow \bar{\partial}_{1,1} & & \downarrow \bar{\partial}_{2,1} & & \\
\Omega^{0,2} & \xrightarrow{\partial_{0,2}} & \Omega^{1,2} & \xrightarrow{\partial_{1,2}} & \Omega^{2,2} & \xrightarrow{\partial_{2,2}} & \\
\downarrow \bar{\partial}_{0,2} & & \downarrow \bar{\partial}_{1,2} & & \downarrow \bar{\partial}_{2,2} & &
\end{array}$$

Note that $\partial \circ \overline{\partial} = \overline{\partial} \circ \partial$ and that $\Omega^{i,j} = \overline{\Omega^{j,i}}$. Inspecting $i = 1, j = 0$ reveals that $\Omega^{1,0}$ are the holomorphic 1-forms, and $\Omega^{0,1}$ are the antiholomorphic 1-forms, on our manifold.

THE HODGE DIAMOND

We now inspect the downward cohomology of the dobeault cohomology. We name the quotient groups that it creates

$$H^{i,j}(M) = ker(\overline{\partial}_{(i,j)}) \Big/ Im(\overline{\partial}_{(i,j-1)})$$

We call the dimension of these groups the **hodge numbers**, $h^{i,j} = |H^{i,j}(M)|$. Then we arrange these into **the Hodge diamond**

The hodge diamond is exceptionally useful in algebraic topology as a tool to classify manifolds. The row that each of these hodge numbers are in corresponds to the weight of the represented group. This weight is the order of the forms contained within the cosets that make up each individual $H^{i,j}(M)$.

$$\begin{array}{ccccc}
 & & h^{0,0} & & \\
 & \iddots & \vdots & \ddots & \\
h^{0,n} & \cdots & h^{n/2,n/2} & \cdots & h^{n,0} \\
 & \ddots & \vdots & \iddots & \\
 & & h^{n,n} & &
\end{array}$$

HODGE STRUCTURES

We call the direct sum of all cohomology groups of a particular weight(k), the **hodge structure of weight k**

$$H^k(M, \mathbb{C}) = \bigoplus_{i+j=k} H^{i,j}(M)$$

In the case of hodge structures of weight 1 we know

$$H^1(M, \mathbb{C}) = H^{1,0}(M) \oplus H^{1,0}(M) = H^{1,0}(M) \oplus \overline{H^{1,0}(M)}$$

So we conclude that $H^1(M, \mathbb{C})$ is of even dimension. This must also be true for the lattice subset $H^1(M, \mathbb{Z}) \subset H^1(M, \mathbb{C})$. Thus we identify a torus

$$T = H^{0,1}(M) \Big/ H^1(M, \mathbb{Z})$$

The map from a complex torus to the cohomology groups generated on that torus yields an inverse map and thus we establish a bijection between complex tori and hodge structrues of weight 1

$$T \leftrightarrow H^1(M, \mathbb{C})$$

ACKNOWLEDGEMENTS AND REFERENCES

Thank you to my mentor, Chris Dare, and the DRP committee for the opportunity to work on this project

Voisin, C. (2002). *Hodge Theory and Complex Algebraic Geometry, I.* Cambridge, UK. Cambridge

7 Appendix A: Source Code for Linear Algebra Content

```python
class SystemOfEquations(Scene):
    def construct(self):

        ############## SCENE 1: Title, intro to problem ################

        # Title construction
        connecting_matrices_text = Text('Connecting Algebra to Geometry').shift(3*UP)
        ul=Underline(connecting_matrices_text)
        self.add(connecting_matrices_text, ul)
        self.play(Write(connecting_matrices_text), Create(ul))
        self.wait(2)

        # Intro text
        asked_to_solve = Text('Suppose we are\n asked to solve').scale(0.6).shift(2*UP + 5*LEFT)
        self.add(asked_to_solve)
        self.play(Write(asked_to_solve))

        # Linear equations
        matheqs = MathTex(r'3x + 2y &= 5 \\ 2x + y &= 1').shift(5*LEFT + 0.4*UP)
        self.add(matheqs)
        self.play(Write(matheqs))
        self.wait(2)

        # Arrow visualizing translation of linear equations to matrix
        arrow = Arrow(start=UP, end=DOWN, color=RED).scale(0.6).shift(5*LEFT + 0.8*DOWN)
        matheqs_matrix = MathTex(
            r'\begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix}'
        ).shift(5*LEFT + 2*DOWN)
        self.play(Create(arrow))
        self.play(Uncreate(arrow), Write(matheqs_matrix))
        self.wait(2)


        ################# SCENE 2: Geometrical set up ##############

        # Construct 2d-plane that vectors are going to sit on
        plane = NumberPlane(
            x_range = (-6, 6),
            y_range = (-6, 6),
            x_length=6, y_length=6,
            axis_config={"include_numbers": True},
        )
        self.add(plane)
        self.play(Create(plane))


        # Standard basis vectors for plane, colored in green in orange
        e1 = Vector([1, 0], color=GREEN, stroke_width=25).scale(0.5)
        e2 = Vector([0, 1], color=ORANGE, stroke_width=25).scale(0.5)
        self.add(e1,e2)
        self.play(Create(e1), Create(e2))
        self.wait(3)


        # Briefly color matrix red and enlarge it, giving the notion that we are
        # somehow clicking on or applying the matrix
        matheqs_matrix1 = MathTex(
            r'\begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix}',
            color=RED
        ).shift(5*LEFT + 2*DOWN).scale(1.5)
        matheqs_matrix2 = MathTex(
            r'\begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix}'
        ).shift(5*LEFT + 2*DOWN)
```

```
        self.play(Transform(matheqs_matrix, matheqs_matrix1))
        self.play(Transform(matheqs_matrix, matheqs_matrix2))

        # Apply the actual transform to the plane
        self.play(ApplyMatrix([[3, 2], [2, 1]], plane),
                  ApplyMatrix([[3, 2], [2, 1]], e1),
                  ApplyMatrix([[3, 2], [2, 1]], e2))
        self.wait(3)



        ################# SCENE 3: Translating the problem from equations to geometric setting ############

        # Move equations out of way
        matheqs_red = MathTex(r’3x + 2y &= 5 \\ 2x + y &= 1’).shift(5*LEFT + 0.4*UP)
        matheqs_red[0][6].set_color(RED)
        matheqs_red[0][12].set_color(RED)
        self.play(Transform(matheqs, matheqs_red))


        # Represent the solution to our linear equations as a vector
        new_vect = Vector( [9, 1], color=RED).scale(0.5).shift(2*LEFT + 0.2*DOWN)
        self.add(new_vect)
        self.play(Create(new_vect))

        new_vect_label = new_vect.coordinate_label(color=RED)
        self.add(new_vect_label)
        self.play(Write(new_vect_label))
        self.wait(2)


        #
        same_as_asking_text = Tex(
            r"This is the same as asking\newline ‘‘What vector $\begin{bmatrix}x \\ y \end{bmatrix}$ got\newline sent to $\begin{bmatrix} 5 \\ 1 \end{bmatrix}$?’’"
        ).scale(0.7).shift(4*RIGHT + 2*DOWN)
        self.add(same_as_asking_text)
        self.play(Write(same_as_asking_text))
        self.wait(7)
```

The above code can be run (after downloading Manim, see https://www.manim.community/) by running

```
manim -pqm SystemOfEquations
```

8 Appendix B: Source Code for Surface of Revolution

It is highly recommended to run this code with the

```
--disable_caching
```

option since several of the helper functions need to be optimized.

```python
from manim.utils.color import Colors
import random

"""
Helper function which generates a random color and translates it into a hexidecimal string

No input
returns: random string of the format #------ where the 6 characters following the # are hexidecimal
"""
def random_color_str():
    # generate random number between #000000 and #FFFFFF
    rand_color = hex(random.randrange(0, 2**24))
    # We want the \# symbol to be included
    rc_str = "#" + str(rand_color[2:])

    # The string must be length 7 (i.e. 6 hexidecimal base numbers and one \# symbol)
    # However, random.randrange will occasionally generate a number too small
    while len(rc_str) < 7:
        rc_str = rc_str + "0"

    return rc_str


"""
Helper function to generate an array of n=num_cyl VGroup objects each containing 2 Surface objects:
    (1) Corresponding to the wall / side of a shell
    (1) Corresponding to a cap of the shell, so the surface of revolution does not appear hollow
which, once displayed, provide a 3D model of our surface of revolution

vars:
    function = a lambda function of a single input variable which represents the underlying f(x) that is being rotated
    axes = the ThreeDAxes object that the surfaces are to be added to
    x_min = a floating point number representing the lower bound on the interval in which the function is being rotated
    x_max = a floating point number representing the upper bound on the interval in which the function is being rotated
    num_cyl = the number of cylinders used to approximate


returns: an array of VGroup objects, each containing 2 surface objects corresponding to a wall and  a cap of the same radius


WARNING: This function is massively inefficient and could use some aggressive optimization
"""
def create_washers_revolution(function, axes, x_min, x_max, num_cyl):

    assert x_min < x_max, "second input (x_min) should be smaller than third input (x_max)"
    assert int(num_cyl) == num_cyl and num_cyl > 0, "num_cyl must be a positive integer"

    # Calculate the width of each cylinder
    step_length = float(x_max - x_min) / num_cyl
```

```
    # initialize the array we will return
    surfaces = []


    # Since there must be at least one cylinder, we inductively begin
    # creating our shells in the desired manner
    rc_str = random_color_str()

    initial_disk = Surface(
            lambda u, v: axes.c2p(
                x_min, v*np.cos(u), v*np.sin(u)
            ),
            u_range=[0, 2*PI], # u represents theta
            v_range=[0, function(x_min)], # v represents our radius
            checkerboard_colors=[rc_str, rc_str]
        )


    # Iteratively begin creating more shells
    for i in range(num_cyl):

        x_val = float(x_min) + i*step_length # increment x position
        f_val = function(x_val) # get corresponding function value at the point

        # We wish to group walls and caps which have the same radius / distance from the
        # center of revolution
        wall_and_cap = VGroup()

        wall = Surface(
            lambda u, v: axes.c2p(
                v, f_val*np.cos(u), f_val*np.sin(u)
            ),
            u_range=[0, 2*PI],  # u represents theta
            v_range=[x_val, x_val + step_length], # v represents the position along the x axis
            checkerboard_colors=[rc_str, rc_str]
        )
        cap = Surface(
            lambda u, v: axes.c2p(
                x_val + step_length, v*np.cos(u), v*np.sin(u)
            ),
            u_range=[0, 2*PI],# u represents theta
            v_range=[0, f_val], # v now represents the radius
            checkerboard_colors=[rc_str, rc_str]
        )
        # Add surfaces to VGroup
        wall_and_cap.add(wall)
        wall_and_cap.add(cap)

        # Add VGroup to
        surfaces.append(wall_and_cap)

        rc_str = random_color_str()


    assert len(surfaces) > 0

    surfaces[0].add(initial_disk)
    return surfaces




class surface_of_rev_washer(ThreeDScene):


    """
```

```
    Helper function to write the title of the video within the first scene
    """
    def produce_title(self, title_str):

        title_text = Text(title_str, color='#5ad2d6')
        ul1 = Underline(title_text, color='#5ad2d6')

        self.add_fixed_in_frame_mobjects(title_text, ul1)
        self.play(Write(title_text))
        self.play(Create(ul1))

        self.wait(2)
        # Remove title for next scene
        self.play(Uncreate(title_text), Uncreate(ul1))



    def construct(self):

        ########### SCENE 1: Print title, get value trackers ############
        phi, theta, focal_distance, gamma, distance_to_origin = self.camera.get_value_trackers()
        axes = ThreeDAxes()

        self.produce_title('Surfaces of Revolution: Washer')



        self.play(Create(axes))
        self.wait(1)

        ############ SCENE 2: Construct axes and setup underlying function ##########

        # Begin to rotate camera
        self.play(phi.animate.increment_value(60*DEGREES),
                  theta.animate.increment_value(30*DEGREES))

        self.wait(1)

        graph = axes.plot(lambda x: (0.25*x**2 + 1), x_range=[0,4], color=YELLOW_A)
        area = axes.get_area(graph=graph, x_range=[0,4], color=YELLOW_E)

        self.play(Create(graph))
        self.wait(1)

        # highlight the area under graph
        self.play(Create(area))
        self.wait(1)


        # Begin to rotate the function 360 degrees around the axis of revolution
        self.play(
            Rotating(
                VGroup(graph, area),
                axis=RIGHT,
                radians=2*PI,
                about_point=axes.c2p(0,0,0)
            ),
            run_time=5,
            rate_func=linear
        )


        ############## SCENE 3: Construct the resulting surface of revolution ###########

        desired_surface = Surface(
            lambda u, v: axes.c2p(
```

```
                v, (0.25*v**2 + 1)*np.cos(u), (0.25*v**2 + 1)*np.sin(u)
            ),
            u_range=[0, 2*PI],
            v_range=[0, 4],
            checkerboard_colors=[YELLOW, YELLOW_E]
        )
        # Add a disk to the top of hte cylinder to give the impression that
        # the shape is not hollow
        desired_surface_cap = Surface(
            lambda u, v: axes.c2p(
                4, v*np.cos(u), v*np.sin(u)
            ),
            u_range=[0, 2*PI],
            v_range=[0, 5],
            checkerboard_colors=[YELLOW, YELLOW_E]
        )


        self.play(Create(desired_surface),
                  Create(desired_surface_cap),
                 run_time=3)
        self.wait(1)

        # Write text in scene
        what_is_volume_text = Text('What is the volume\n of this shape?').scale(0.6).shift(3*LEFT + 3*UP)
        self.add_fixed_in_frame_mobjects(what_is_volume_text)
        self.play(Write(what_is_volume_text),
                 run_time=2)
        self.wait(3)

        self.play(Uncreate(what_is_volume_text),
                  Uncreate(desired_surface),
                  Uncreate(desired_surface_cap))
        self.wait(1)



        ################# SCENE 4: Demonstration of construction of shell ##############

        use_familiar_shapes_text = Text('Idea: use familiar shapes\n like cylinders to approximate').scale(0.6).shift(4*LEFT + 3*UP)
        self.add_fixed_in_frame_mobjects(use_familiar_shapes_text)
        self.play(Write(use_familiar_shapes_text),
                  run_time=2)

        # Construct a rectangle of width 0.5 under the graph of our function
        line1 = Line(
                start=axes.c2p(2, 0),
                end=axes.c2p(2, graph.underlying_function(2)),
                stroke_color=GREEN
            )
        self.play(Create(line1))
        line2 = Line(
                start=axes.c2p(2.5, 0),
                end=axes.c2p(2.5, graph.underlying_function(2)),
                stroke_color=GREEN
            )
        line3 = Line(start=axes.c2p(2, graph.underlying_function(2)),
                     end=axes.c2p(2.5, graph.underlying_function(2)),
                     stroke_color=GREEN
                    )
        self.play(Create(line2), Create(line3))
        self.wait(1)


        # Begin to rotate the rectangle 360 degrees around the axis of revolution to
```

```
give
        # The impression of constructing a disk
        self.play(
            Rotating(
                VGroup(line1, line2, line3),
                axis=RIGHT,
                radians=2*PI,
                about_point=axes.c2p(0,0,0)
            ),
            run_time=2,
            rate_func=linear
        )




        # Fill in the area swept out by rotating the rectangle with an
        # actual cylinder. However, the cylinder should not appear hollow
        # So we must add a disk to the top and bottom to make it look filled
        # in
        cyl_cap1 = Surface(
            lambda u, v: axes.c2p(
                2, v*np.cos(u), v*np.sin(u)
            ),
            u_range=[0, 2*PI],
            v_range=[0, 2],
            checkerboard_colors=[GREEN, YELLOW_E]
        )
        cyl_cap2 = Surface(
            lambda u, v: axes.c2p(
                2.5, v*np.cos(u), v*np.sin(u)
            ),
            u_range=[0, 2*PI],
            v_range=[0, 2],
            checkerboard_colors=[GREEN, GREEN_E]
        )
        cyl_wall1 = Surface(
            lambda u, v: axes.c2p(
                v, 2*np.cos(u), 2*np.sin(u)
            ),
            u_range=[0, 2*PI],
            v_range=[2, 2.5],
            checkerboard_colors=[GREEN, GREEN_E]
        )

        self.play(Create(cyl_cap1),
                  Create(cyl_cap2),
                  Create(cyl_wall1),
                 run_time=2)

        self.wait(1)


        # Provide formula for volume of this "solid"
        area_cyl_text = Text('Volume( cylinder ) =', t2c={'cylinder' : GREEN}).scale(0.6).shift(4.2*LEFT)
        self.add_fixed_in_frame_mobjects(area_cyl_text)
        self.play(Write(area_cyl_text))

        formula_cyl_tex = Tex(r'$\pi( \text{radius} )^2 \times \text{width}$').shift(4.8*LEFT+0.6*DOWN)
        self.add_fixed_in_frame_mobjects(formula_cyl_tex)
        self.play(Write(formula_cyl_tex))

        self.wait(2)

        the_radius_in_this_text = Text('The radius in this case is just\n the y-coordinate of y = f(x)').scale(0.6).shift(4.4*LEFT+3*DOWN)
```

```
        self.add_fixed_in_frame_mobjects(the_radius_in_this_text)
        self.play(Write(the_radius_in_this_text))
        self.wait(1)

        formula_cyl_tex_new = Tex(r'$\pi( f(x) )^2 \times \text{width}$').shift(4.8*LEFT+0.6*DOWN)
        self.play(Uncreate(formula_cyl_tex))
        self.add_fixed_in_frame_mobjects(formula_cyl_tex_new)
        self.play(Write(formula_cyl_tex_new))

        # Clean up scene
        self.wait(2)
        self.play(Uncreate(the_radius_in_this_text), Uncreate(formula_cyl_tex_new), Uncreate(area_cyl_text))

        now_repeat_to_fill_in_text = Text('Now repeat until the shape is filled in').scale(0.6).shift(4*LEFT+3*DOWN)
        self.add_fixed_in_frame_mobjects(now_repeat_to_fill_in_text)
        self.play(Write(now_repeat_to_fill_in_text))

        self.wait(1)

        ############## SCENE 5: Use multiple shells to approximate volume #############


        # TODO: Replace the code below with call to create_washers_of_revolution


        # Create 6 shells to fill in the region from x_min to x_max
        cyl_wall2 = Surface(
            lambda u, v: axes.c2p(
                v, 1.5625*np.cos(u), 1.5625*np.sin(u)
            ),
            u_range=[0, 2*PI],
            v_range=[1.5, 2],
            checkerboard_colors=[TEAL, TEAL_E]
        )

        cyl_cap6 = Surface(
            lambda u, v: axes.c2p(
                3, v*np.cos(u), v*np.sin(u)
            ),
            u_range=[0, 2*PI],
            v_range=[0, 2.5625],
            checkerboard_colors=[BLUE, BLUE_E]
        )
        cyl_wall3 = Surface(
            lambda u, v: axes.c2p(
                v, 2.5625*np.cos(u), 2.5625*np.sin(u)
            ),
            u_range=[0, 2*PI],
            v_range=[2.5, 3],
            checkerboard_colors=[BLUE, BLUE_E]
        )
        # Create first and second shell
        self.play(Create(cyl_wall2),
                  Create(cyl_cap6),
                  Create(cyl_wall3))



        cyl_wall4 = Surface(
            lambda u, v: axes.c2p(
                v, 1.25*np.cos(u), 1.25*np.sin(u)
            ),
            u_range=[0, 2*PI],
            v_range=[1, 1.5],
```

```
            checkerboard_colors=[MAROON, MAROON_E]
        )

        cyl_cap10 = Surface(
            lambda u, v: axes.c2p(
                3.5, v*np.cos(u), v*np.sin(u)
            ),
            u_range=[0, 2*PI],
            v_range=[0, 3.25],
            checkerboard_colors=[PURPLE, PURPLE_E]
        )
        cyl_wall5 = Surface(
            lambda u, v: axes.c2p(
                v, 3.25*np.cos(u), 3.25*np.sin(u)
            ),
            u_range=[0, 2*PI],
            v_range=[3, 3.5],
            checkerboard_colors=[PURPLE, PURPLE_E]
        )
        # Create third and fourth shell
        self.play(Create(cyl_wall4),
                  Create(cyl_cap10),
                  Create(cyl_wall5))



        cyl_wall6 = Surface(
            lambda u, v: axes.c2p(
                v, 1.06*np.cos(u), 1.06*np.sin(u)
            ),
            u_range=[0, 2*PI],
            v_range=[0.5, 1],
            checkerboard_colors=[RED, RED_E]
        )

        cyl_cap14 = Surface(
            lambda u, v: axes.c2p(
                4, v*np.cos(u), v*np.sin(u)
            ),
            u_range=[0, 2*PI],
            v_range=[0, 4.06],
            checkerboard_colors=[PINK, PURPLE_A]
        )
        cyl_wall7 = Surface(
            lambda u, v: axes.c2p(
                v, 4.06*np.cos(u), 4.06*np.sin(u)
            ),
            u_range=[0, 2*PI],
            v_range=[3.5, 4],
            checkerboard_colors=[PINK, PURPLE_A]
        )
        # Create fifth and sixth shell
        self.play(Create(cyl_wall6),
                  Create(cyl_cap14),
                  Create(cyl_wall7))

        self.wait(2)
        self.play(Uncreate(now_repeat_to_fill_in_text))


        # Write down relevant equations
        total_volume_tex = Tex(r'$\text{Volume}\approx $').scale(0.8).shift(4.5*LEFT)
        total_volume_formula_tex = Tex(r'$\sum_i \pi \times f(x_i)^2 \times \text{
    width}$').scale(0.8).shift(4.4*LEFT+DOWN)
        self.add_fixed_in_frame_mobjects(total_volume_tex, total_volume_formula_tex)
        self.play(Write(total_volume_tex),
                 Write(total_volume_formula_tex))
        self.wait(3)
```

```


        # Clean up scene
        self.play(Uncreate(cyl_wall1),
                  Uncreate(cyl_wall2),
                  Uncreate(cyl_wall3),
                  Uncreate(cyl_wall4),
                  Uncreate(cyl_wall5),
                  Uncreate(cyl_wall6),
                  Uncreate(cyl_wall7),
                  Uncreate(cyl_cap1),
                  Uncreate(cyl_cap2),
                  Uncreate(cyl_cap6),
                  Uncreate(cyl_cap10),
                  Uncreate(cyl_cap14))


        ################## SCENE 6: Increase number of shells to closer approximate ###########

        this_approx_text = Tex(r'This approx. becomes more accurate as we take limit Width $\to 0$').scale(0.6).shift(3.4*DOWN)
        self.add_fixed_in_frame_mobjects(this_approx_text)
        self.play(Write(this_approx_text))

        self.wait(1)


        # TODO: somehow speed up performance here
        surfaces = create_washers_revolution(lambda x : (0.25*x**2 + 1), axes, 0, 4, 16)

        for surface in surfaces:
            self.play(Create(surface))

        self.wait(1)
```